Kamakura Corporation

Fair Value and Expected Credit Loss Estimation: An Accuracy Comparison of Bond Price versus Spread Analysis Using Lehman Data

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The International Financial Reporting Standard ("IFRS") 9 and the Financial Accounting Standard Board's ("FASB") Current Expected Credit Loss ("CECL") model significantly raise the accuracy bar for valuation and credit risk analytics for all organizations who report under their aegis. In both cases, the visibility of the organization's valuation and credit risk assessment moves from the back office or middle office, seen primarily by risk experts, to center stage under a bright spot light. Both these standards allow the use of creditworthiness assessment using approaches encompassing:

- Probabilities of default
- Internal or external credit ratings
- Credit spreads

The objective of the standards is the generation of 12-month and expected lifetime loan losses based on changes to obligor creditworthiness from one observed point to the next.

It is to the last bullet point above that we focus our attention, and in this note, take a model validation approach and compare two methods of valuation and credit loss assessment from an accuracy point of view. The first approach uses market-based credit spreads to establish obligor creditworthiness, estimate values and credit losses. The second approach uses observable market prices of securities, rather than credit spreads derived from them, directly in the valuation and credit assessment process. We explain why the use of observable market prices is best practice. We also list the model validation issues that cause credit spreads (derived from market prices) to be a source of random errors in valuation and credit assessment.

Which Market? The Bond Market or the Credit Default Swap Market?

What is the best source of market data, whether it be securities prices or credit spreads derived from them, for any creditworthiness assessment, including IFRS 9 and CECL expected loss calculations? In a perfect world, the simple answer would be "all markets." Sadly, there is a substantial imbalance in the transparency and price discovery available in two key markets that are potentially important for IFRS 9 and CECL: the corporate bond market in the United States and the market for single name credit default swaps.

The chart below presents the trading volume and most heavily traded reference names in the U.S. corporate bond market on a representative day, April 19, 2016:

Kamakura Corporation
Top 20 Bond Issuers by Daily Volume Traded
Corporate Fixed Rate Bond Trading Volume for
April 19, 2016
Source: Kamakura Corporation, TRACE, Market Axess

Rank	Reference Name		Bond Trading Volume (U.S. Dollars)	Number of Trades	Number of Issues Traded
Total Number of Issuers		**1290**	**11,441,469,440**	**33,680**	**4,722**
1	MORGAN STANLEY		556,794,852	925	147
2	BANK OF AMERICA CORP		522,013,615	1,127	154
3	WELLS FARGO & CO		384,073,000	749	50
4	VERIZON COMMUNICATIONS INC		304,326,000	391	30
5	CITIGROUP INC		302,351,000	840	68
6	JPMORGAN CHASE & CO		228,787,300	691	62
7	PETROBRAS GLOBAL FINANCE BV		219,741,000	599	22
8	PETROLEOS MEXICANOS		193,409,000	186	23
9	VALE OVERSEAS LTD		184,775,000	341	7
10	ROYAL BANK OF CANADA		136,624,730	222	40
11	GOLDMAN SACHS GROUP INC		135,864,790	1,094	161
12	HSBC HOLDINGS PLC		113,362,000	142	11
13	BARCLAYS PLC		109,018,000	98	8
14	GENERAL ELECTRIC CO		107,001,000	798	137
15	UNITEDHEALTH GROUP INC		106,923,000	149	24
16	APPLE INC		100,046,000	399	28
17	FORD MOTOR CREDIT COMPANY LLC		99,715,000	128	28
18	BANK OF NOVA SCOTIA		98,985,000	157	21
19	AT&T INC		86,750,000	368	31
20	TARGET CORP		84,210,000	88	13

The data is provided by the Financial Industry Regulatory Authority ("FINRA") via the Trade Reporting and Compliance Engine ("TRACE") system which records every trade in the U.S. corporate bond market and the prices at which trades took place. On April 19, there were 33,680 bond trades on 4,722 bonds issued by 1,290 bond issuers for an underlying principal amount of $11.4 billion.

The volume of single name credit default swaps traded, but not the spreads or prices, has been reported by the Depository Trust & Clearing Corporation since July, 2010. Data is reported weekly with a four calendar day lag, not daily. Trading volume for the week ended April 15 is given in the chart below:

Kamakura Corporation
Top Reference Names by Notional Principal Traded
Credit Default Swap Trading Volume for Week Ended
April 15, 2016
Source: Kamakura Corporation, Depository Trust & Clearing Corporation

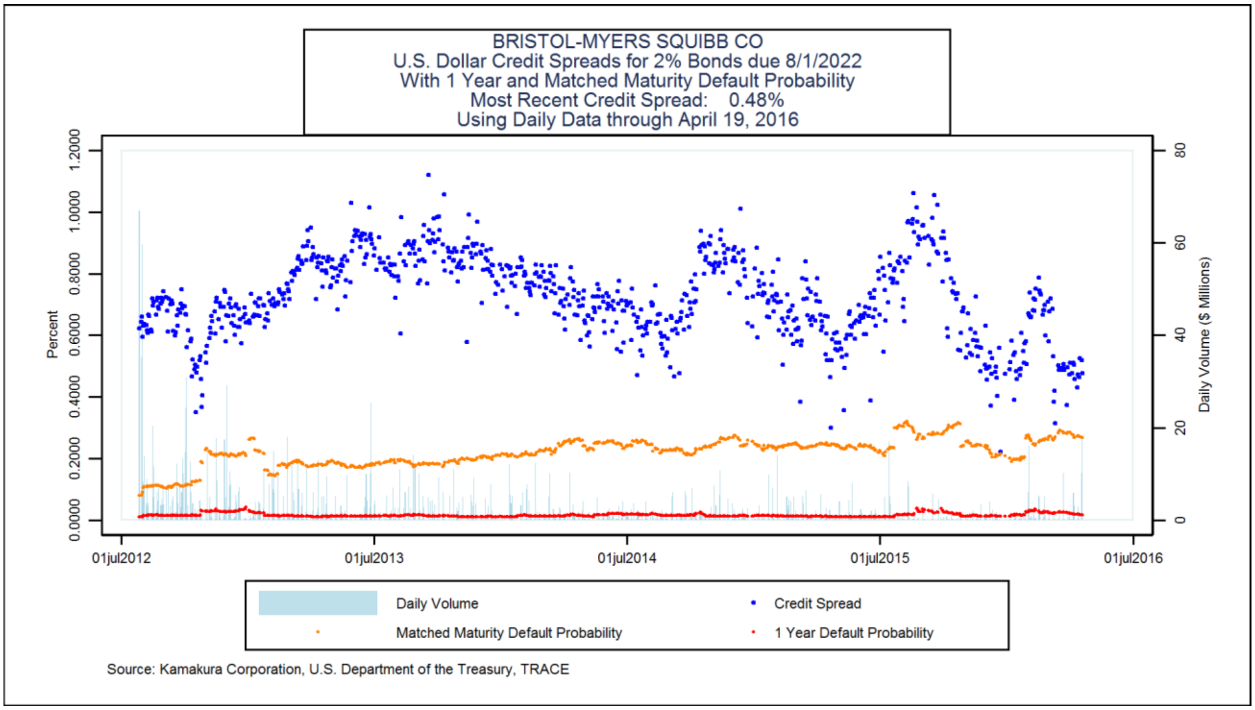
	Reference Names	Notional Principal Traded (U.S. Dollars)	Number of Trades
Totals:	**734**	**56,248,033,477**	**14,000**

Rank	Reference Name	Notional Principal Traded (U.S. Dollars)	Number of Trades
1	FEDERATIVE REPUBLIC OF BRAZIL	3,242,905,600	429
2	RUSSIAN FEDERATION	1,618,562,304	178
3	PEOPLES REPUBLIC OF CHINA	1,551,779,968	261
4	REPUBLIC OF SOUTH AFRICA	1,356,872,576	223
5	REPUBLIC OF TURKEY	1,310,817,920	141
6	REPUBLIC OF PERU	1,234,265,984	209
7	**VOLKSWAGEN AKTIENGESELLSCHAFT**	787,623,104	143
8	UNITED MEXICAN STATES	776,275,008	144
9	REPUBLIC OF KOREA	757,849,984	58
10	REPUBLIC OF THE PHILIPPINES	677,500,032	50
11	**BERKSHIRE HATHAWAY INC.**	594,350,016	52
12	FRENCH REPUBLIC	569,200,000	184
13	**AMERICAN EXPRESS COMPANY**	551,049,984	129
14	UNITED KINGDOM OF GREAT BRITAIN AND NORTHERN IRELAND	474,507,392	11
15	REPUBLIC OF INDONESIA	452,049,984	135
16	**ANADARKO PETROLEUM CORPORATION**	427,960,000	77
17	REPUBLIC OF ITALY	419,400,000	36
18	**BARRICK GOLD CORPORATION**	419,200,000	64
19	**MACYS, INC.**	408,400,000	49
20	**DEUTSCHE BANK AKTIENGESELLSCHAFT**	378,805,024	107

The trading volume during the week ended April 15 was $56.2 billion for 5 business days, or $11.2 billion per day. While this is comparable to the daily volume in the U.S. corporate bond market, trading volume is dominated by trades in sovereign reference names at the top of the volume ranking, as the chart shows. The number of reference names traded in a week, at 734, is more than 500 fewer than the daily number of reference names traded in the U.S. bond market. The number of trades, at 14,000 per week or 2,800 per day, is less than one-tenth of the U.S. corporate bond market trade count. As mentioned above, the Depository Trust and Clearing Corporation does not report actual traded spreads or prices. **In addition, the leading vendors of credit default swap data report quotes, not traded spreads or prices.** Kamakura Corporation's estimates, using the volume numbers above, are that less than 3% of the credit spreads reported by these data vendors could possibly be associated with real trades, and that 3% is not identified by the vendor. In other words, reliance on anonymous quotes from Wall Street is certainly not a reliable basis for any expected loss calculations because of the obvious conflict of interest that the anonymous dealers have.

A Lehman Brothers Example

Valuation and credit assessment for IFRS 9 and CECL for a riskless borrower is quite simple. The analysis becomes progressively more difficult to do accurately as the credit risk of the obligor increases. The impact of non-default probability factors on credit spreads, however, gives credit spreads a much higher volatility than the matched maturity default probability (in orange) and one year default probability (in red). This volatility differential is very visible in this graph of credit spreads (in blue), matched maturity default probabilities (in orange), and 1 year default probabilities (in red) on the Bristol-Myers Squibb bond due 2022:



Given the fact that spread modeling is even more complex for high default risk issuers, it is instructive to do our model validation using data for the now bankrupt Lehman Brothers. Lehman Brothers Holdings Inc. announced its intention to file for bankruptcy on Sunday, September 14, 2008. Bond prices and spreads on Monday, September 15, 2008, fully reflect this information. We use this data from TRACE on 22 senior non-call bonds that traded at least $5 million in daily volume on that day:

LEHMAN BROTHERS HOLDINGS INC

Filter By: Entity All | Type Fixed | Currency USD | Optionality Non-Callable | Seniority Senior

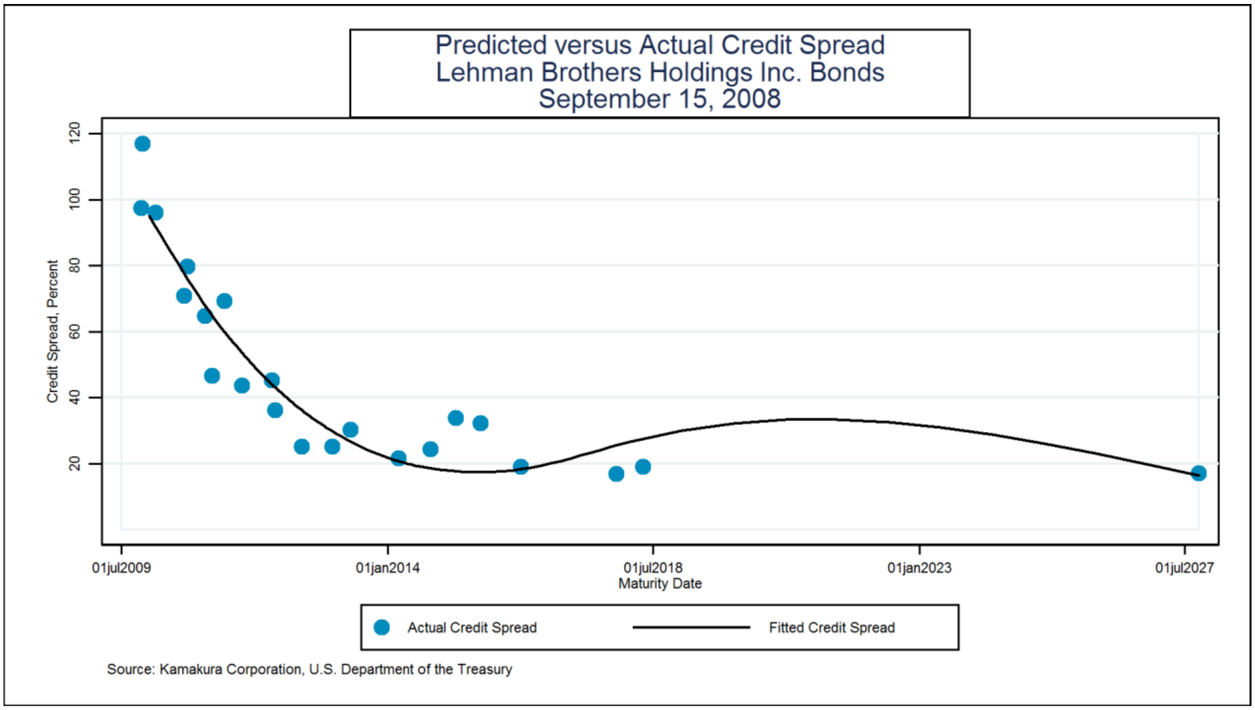
ISIN	Security Description	Issuer	Volume	Price	Yield	Spread
US5252M0FD44	6.875% MTN REDEEM 02/05/2018 USD 1000 - I	Lehman Brothers Holdings Inc.	378,908,000	35.6255	22.3786	18.9682
US5252M0BZ91	5.625% MTN REDEEM 24/01/2013 USD 1000	Lehman Brothers Holdings Inc.	244,121,000	35.0512	27.4539	25.0496
US52517PF635	5.50% MTN REDEEM 04/04/2016 USD 1000	Lehman Brothers Holdings Plc	177,292,000	34.4464	22.0458	18.9673
US52517P5X54	6.20% MTN REDEEM 26/09/2014 USD 1000	Lehman Brothers Holdings Inc.	143,807,000	35.0301	27.1185	24.3225
US52517PSC67	6.625% NT REDEEM 18/01/2012 USD	Lehman Brothers Holdings.	109,940,000	35.4192	47.3425	45.2333
US52517P4C27	6% NT REDEEM 19/07/2012 USD 1000	Lehman Brothers Holdings Inc.	107,493,000	34.4558	27.4610	25.2066
US52517PH615	5.75% MTN REDEEM 17/05/2013 USD 1000	Lehman Brothers Holdings Inc.	104,551,000	32.3261	32.7225	30.2285
US52517PR606	5.25% MTN REDEEM 06/02/2012 USD 1000	Lehman Brothers Holdings Inc.	97,282,000	35.9846	38.3629	36.2386
US524908CM04	7.875% NT REDEEM 15/08/2010 USD 1000	Lehman Brothers Holdings.	63,256,000	34.1937	81.4908	79.7210
US52517PD572	5% MTN REDEEM 14/01/2011 USD 1000	Lehman Brothers Holdings Inc.	51,455,000	32.2662	48.3974	46.5412
US52517PVV02	4.80% MTN REDEEM 13/03/2014 USD 1000	Lehman Brothers Holdings Inc.	45,343,000	34.1998	24.3567	21.6686
US52517PA354	4.50% MTN REDEEM 26/07/2010 USD 1000	Lehman Brothers Holdings Inc.	40,897,000	34.7447	72.6557	70.8925
US52517PXT38	3.95% MTN REDEEM 10/11/2009 USD 1000	Lehman Brothers Holdings.	36,945,000	34.1356	118.6399	116.9615
US52517PYN58	4.25% MTN REDEEM 27/01/2010 USD 1000	Lehman Brothers Holdings.	33,244,000	33.5059	97.7935	96.0894
US52517PVM03	4.375% NT REDEEM 30/11/2010 USD 1000 - Ser `G`	Lehman Brothers Holdings Inc.	30,558,000	33.4258	66.4639	64.6360
US52517PK593	5.75% NT REDEEM 18/07/2011 USD 1000	Lehman Brothers Holdings Inc.	28,577,000	32.7110	45.5670	43.5942
US524908CF52	7.875% NT REDEEM 01/11/2009 USD 1000	Lehman Brothers Holdings.	27,126,000	33.7391	98.9661	97.2906
US524908AA83	8.80% NT REDEEM 01/03/2015 USD 1000	Lehman Brothers Holdings.	25,913,000	32.7536	36.7308	33.8493
US52517PSZ52	5.875% MTN REDEEM 15/11/2017 USD 1000	Lehman Brothers Holdings Inc.	19,181,000	34.1538	20.2422	16.9053
US524908BF61	8.50% NT REDEEM 01/08/2015 USD 1000	Lehman Brothers Holdings.	15,232,000	32.3032	35.1887	32.2234
US52517P5Y38	7% MTN REDEEM 27/09/2027 USD 1000	Lehman Brothers Holdings.	6,887,000	34.5306	21.1810	17.1059
US82087KAG40	6% NT REDEEM 01/04/2011 USD	Lehman Brothers Holdings Inc.	5,000,000	28.5000	71.1316	69.2268

The credit spreads on these issues ranged from 16.91% to 116.96%, depending on the maturity of the bond, among other things. The credit spreads are graphed by maturity date in the figure below:

With this data in hand, we now turn to a discussion of common practice and best practice using either bond prices or credit spreads as input.

Common Practice: Model Validation Using Credit Spreads

In the graph above, the credit spreads on Lehman Brothers Holdings Inc. senior non-call bonds had an average credit spread of 46.86% with a standard deviation of 29.73% on September 15, 2008. A common practice approach to the computation of expected losses using credit spreads ascribes the variation in spreads to noise in the bond price data, and the analyst therefore “solves” this problem by fitting a smooth function to the Lehman Brothers spread data. The result is a commonly observed downward sloping spread as a function of years to maturity:



In the graph above, we have used a cubic function of years to maturity because of the common use of cubic splines in yield curve smoothing in general. At first glance, the fitting seems successful, because the three terms in the regression (years to maturity, years squared and years cubed) are all statistically significant and the adjusted r-squared in the regression is a respectable 88.49%. A red flag appears, however, in the form of a root mean squared error of the regression with a value of 10.09%, meaning the errors in the regression have a standard deviation of 10 full percentage points. Should that be worrisome?

We can address that question from a model validation perspective by answering a more basic question about the mathematical formula for the credit spread. For a bond with semi-annual payments, a semi-annual coupon of C dollars, and a principal amount of $100 due in exactly n semi-annual periods, the value of the credit spread s (expressed as a decimal) is the constant such that this formula equals the bond’s net present value V, which is the sum of price P and accrued interest A (which will be zero in this example):

$$V = P + A = C \sum_{i=1}^{n} \frac{1}{\left[1 + \frac{r}{2} + \frac{s}{2}\right]^i} + \frac{100}{\left[1 + \frac{r}{2} + \frac{s}{2}\right]^n}$$

We can say that the implied annualized value of the spread s is a function of the periods to maturity n, the matched maturity risk free yield on U.S. Treasuries r (expressed as a decimal), and the bond's semi-annual dollar coupon C and value V (or alternatively price P and accrued interest A):

$$s = f(n, r, C, V)$$

We have learned a lot from this exercise, and all of it is troubling. First, the cubic polynomial we have fitted above is mis-specified, because we have omitted the risk free Treasury yield level r, the bond's semi-annual dollar coupon C, and its net present value V. Because of these omitted variables, we have caused our original Lehman credit spreads to be LESS accurate than they were before. The original credit spreads, by definition, were 100% consistent with the original bond prices, because they were derived from the equation for net present value V. The new fitted spreads no longer match observed bond prices.

Sadly, there are many more problems with the equation above, because the equation involves a large number of assumptions which we now know to be wrong in the case where default risk is not zero. Using Lehman Brothers data as an extreme example makes these false assumptions clear:

1. The corporate bond will pay its full principal amount (this argument is false: the bond is defaulting and will pay its recovery value). In the Lehman case, the average bond price is 33.80, with a relatively small standard deviation of 1.60 over the 22 bond issues. If we say that the recovery amount is roughly 33.80, the assumption that the bond will pay 100 is grossly wrong and overstated.
2. The full principal amount will be paid at maturity (false: the recovery amount will be paid upon resolution of bankruptcy proceedings in court. The longest maturity bond from Lehman in the chart above is 2027, but most of the recovery payments to Lehman bond holders have already been made).
3. All interest coupons will be paid (false: only those interest payments prior to the bankruptcy filing on September 15, 2008 will be paid).
4. Bonds of different maturities and coupons have different cash flows (false: they have identical cash flows upon default: interest payments are zero and the principal that will be paid is the recovery amount; and the payment date is the date [or series of dates] that recovery payments are made after the bankruptcy is resolved in court).
5. Credit spreads are constant for all periods prior to maturity of bond k (false, they vary by maturity for firms that are not near bankruptcy).
6. Credit spreads for bond k are different from bond j if they have different maturities, but these constant spreads are inconsistent from time zero to years to maturity = min(j,k). To give a specific example, the credit spread formula implies that the

credit spread is 16.96% for the 2027 bond but 45.23% for the bond due in January 2012. In short, for the period from September 2008 to January 2012, the spread formula implies that the coupons for the 2012 bond have a spread that is almost 30 percentage points higher than the 16.96% spread that applies to coupons covering the same time period on the bond due in 2027. This inconsistency is nonsense.

7. The risk free yield is constant for all periods until the risk-free bond's maturity (false, this is a well-known problem with the yield to maturity calculation). Even for the risk free curve, the yield to maturity for bonds of different maturities implies different discount rates during the overlapping period when both bonds are outstanding.

After reviewing the model validation errors in the credit spread formula, one comes to a horrible realization: the "noise" in the credit spread graph above is not due to the noise in bond prices *per se*; instead the noise is due to omitted variables in the spread formula and false assumptions that are relevant whenever the default probability of the bond issuer is not zero (note that issues 6 and 7 apply even if the default probability is zero).

The result of all of these issues is simple to summarize: by "cleansing" noisy spread data that is in fact due to false model assumptions, the resulting predicted bond prices will be much more volatile than the original bond data itself. Our conclusion is inescapable: we reject the use of traditional credit spreads as an intermediate calculation in any obligor valuation and credit assessments.

Best Practice: Model Validation Using Bond Prices and No Arbitrage Assumptions

We now know that the original bond prices for Lehman have far less "noise" in them than bond prices predicted by smoothing flawed credit spreads. We plot the original bond prices here with the addition of one outlier where the trading volume is $48,000 on September 15 instead of the floor we imposed of $5 million trading volume:

As mentioned above, the average price of the bonds as they converge to the market's perceived recovery value is 33.80 with a standard deviation of 1.60. We have four questions:

1. How do we estimate the price for the outlier bond which the price of 72 is just "bad data" due to small volume and a disadvantaged panic in execution?
2. How do we estimate the price of other Lehman bonds for which there were no trades?
3. How do we estimate all bond prices for a less distressed firm, ABC Brothers, if only some of ABC Brothers bonds are observable?
4. How do we estimate all bond prices for ABC Brothers if some of the bonds are callable?

The pricing of securities whose value depends on a number of macro factors (like the Bristol-Myers Squibb bonds above) in an environment where multiple factors drive the risk-free yield curve was described by Amin and Jarrow [1992], as modified for default risk by Jarrow [2013]. Detailed technical guides (Kamakura Corporation, 2015 and 2016) discuss the no arbitrage valuation for discrete Monte Carlo simulation of the risk free yield curve, macro factors, and default probabilities. Both risk free yields and credit spreads are smoothed while analyzing all bonds of ABC Brothers jointly. Best practice valuation has these characteristics:

- The risk free zero coupon bonds vary by maturity and apply to all of the bonds of ABC Brothers in a consistent way.

- The defaultable bonds of ABC Brothers comprise a series of "building block" securities that apply to all bonds in a consistent manner. Assume that there are 3 bonds outstanding which mature in 2, 4, and 6 semi-annual periods. The primitive securities consist of these 9 building blocks:

I. A security (which represents a coupon payment) which pays $1 in 1 period if default has not occurred before that time
II. A security which pays $1 in 2 periods if default has not occurred before that time
III. A security which pays $1 in 3 periods if default has not occurred before that time
IV. A security which pays $1 in 4 periods if default has not occurred before that time
V. A security which pays $1 in 5 periods if default has not occurred before that time
VI. A security which pays $1 in 6 periods if default has not occurred before that time
VII. A security (which represents principal) and pays $1 in 2 periods if default has not occurred before then and which pays a random recovery D at the time of default if default occurs before period 2.
VIII. A security (which represents principal) and pays $1 in 4 periods if default has not occurred before then and which pays a random recovery D at the time of default if default occurs before period 4
IX. A security (which represents principal) and pays $1 in 6 periods if default has not occurred before then and which pays a random recovery D at the time of default if default occurs before period 6

- Let the three bonds of ABC Brothers have coupons of C[1], C[2] and C[3] dollars per semi-annual period and have a principal amount of 100. Let the value of the 9 building block securities be W[1], W[2],…W[9]. Then the net present value V[1], V[2] and V[3] of the 3 ABC Brothers bonds are the sum of the interest and principal parts:

$$V[1] = C[1](W[1]+W[2])+100\ W[7]$$
$$V[2] = C[2](W[1]+W[2]+W[3]+W[4])+100\ W[8]$$
$$V[3] = C[3](W[1]+W[2]+W[3]+W[4]+W[5]+W[6])+100\ W[9]$$

We also know the values of these building block securities if the real name of “ABC Brothers” is “Lehman Brothers.” If

$W[1]= W[2]= W[3]= W[4]= W[5]= W[6]=0$

and

$W[7]= W[8]= W[9]=0.3380$

then all three bonds will be valued at the average price of Lehman Brothers Holdings Inc. senior non-call debt on September 15, 2008:

$V[1]=V[2]=V[3]=33.80$

Jarrow and Turnbull [1995] were the first to value these building block securities in a random interest rate environment incorporating obligor defaults. Of course, once we have the predicted bond prices (which should be very close to, if not exactly equal to, observable prices) we can then apply the (flawed) credit spread equation above.

The result of this no arbitrage valuation derived using all bond issues of ABC Brothers for which bond prices are observable minimizes the sum of squared pricing errors in a best practice no arbitrage/multi-factor economy. Even when the bonds are callable, the no arbitrage framework of Amin and Jarrow [1992], Heath Jarrow and Morton [1992], and Jarrow [2013] applies.

Conclusion

The common use of credit spreads, which are derived from observable bond prices but which contain false assumptions, contributes large errors to valuation and credit assessment in the obligor creditworthiness analytical process, and this includes obligor creditworthiness estimates based on credit spreads for IFRS9 or CECL expected loss computation processes. We can avoid this incremental source of error by fitting basic building block securities to observable bond prices in a no arbitrage framework. The result is a consistent and highly accurate valuation and credit estimation framework that meets best practice standards of financial theory, econometrics, and trading precision.

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